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A guided wave study of the voltage dependent director configuration in a nematic liquid crystal layer with finite surface tilt

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Prism coupling techniques have been used to excite optical modes in a thin nematic liquid crystal with finite surface tilt in order to study the voltage dependent director profile. The surface tilts are opposite in character and it is found that at zero applied volts the stable configuration is the substantially horizontal state. On applying the field this state is broken, probably transforming to the twisted vertical state. By modelling all the data obtained, the detailed behaviour of the director profile has been fully characterized yielding much information, including the change of surface tilt with applied voltage. For the nematic liquid crystal E7, this gives a voltage induced surface tilt of approximately 0·67° V^{-1} for a 5·65 μm thick cell. Also using a boundary layer model, it has been possible to analyse the free energy in the cell and hence show that the observed twisted vertical state is the expected stable state under the field applied.

1. Introduction

It has been established for some time that a tilted surface alignment of nematic liquid crystals leads to the possibility of different stable director configurations with increasing voltage across the liquid crystal, provided the surface tilts are in the opposite sense [1–3]. (By opposite sense we mean that the vectors, defined as the director leaving the surface, both point in the same x direction, and of course in the opposite z direction, see figure 1.) This has led to the use of such aligned cells in device applications.

There are three possible equilibrium states of the director configuration in this geometry, depending upon the surface director tilt angle, θ_0, measured from the cell surface, and the elastic constants of the material. These are a predominantly horizontal state H, a twisted vertical state T, if the surface tilt angle θ_0 is less than some critical angle θ_c and a planar vertical state V, if the surface tilt angle θ_0 is greater than θ_c. Application of a field for a positively anisotropic liquid crystal will result in a transition from the H to the T (or V) state in the presence of disclinations. The energies, director profile and switching processes for such a cell have been characterized by Thurston and co-workers [4, 5] using a Poincaré sphere description for the director with a boundary layer model. As far as we are aware no detailed guided mode study of these structures has previously been undertaken.

In this study then we utilize the now quite well-established attenuated total reflection technique [6–9] to study the liquid crystal director profile in an asymmetrically surface tilted cell through the excitation of guided modes. We examine the changes in the structure of the director under application of a field by modelling the angular dependent reflectivity from predictions of the director based on the continuum theory [10] together with multilayer optics [11].

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Using the eutectic mixture E7 which is nematic at 40°C in a quite high surface tilt cell, we have first established the existence of the predominately horizontal state at zero volts and then observed the transition to the twisted vertical state under the application of an electric field. Further the free energy of the liquid crystal cell has been analysed showing that, in the field applied, the lower energy state is indeed the observed twisted state.

2. Experimental

A high refractive index ($n = 1{\cdot}8$ at $\lambda = 632{\cdot}8$ nm) equilateral glass prism is coated on one face with a thin layer (~40 nm) of 99·999 per cent pure silver by conventional vacuum evaporation. The optical parameters of this layer are determined by measuring the angular dependent reflectivity of the prism/silver boundary using p-polarized (TM) radiation and fitting the resultant surface plasmon resonance to Fresnel theory [12]. In the current case, this gave for the silver layer at 632·8 nm, $\varepsilon = -18{\cdot}07 + i0{\cdot}63$ with a thickness of 39 nm. On top of this characterized silver layer is deposited, again by vacuum evaporation, an aligning SiO_x layer. In this case, finite surface tilt of the liquid crystal director was required, so the SiO_x was deposited at an angle of 74° to the substrate normal, which is expected to produce a surface director tilt between 3° and 9° [13] measured with respect to the surface of the cell. An optical flat is also coated with an opaque layer of silver and a similar layer of SiO_x to provide the other aligning surface, the backplate. Finally the complete cell of coupling prism, 6 μm thick mylar spacers, and backplate is assembled in a clean room with the aligning surface arranged to give opposite surface tilts. The cell was warmed and then capillary filled with the liquid crystal in the isotropic phase; the complete arrangement is illustrated in figure 1. This sample was then slowly cooled into the nematic phase to give a well-aligned sample.

The liquid crystal filled cell, mounted in an isothermal environment at 40°C, was placed on an optical system which monitors accurately the reflectivity of a 632·8 nm laser beam from the prism/silver/SiO_x/liquid crystal boundary as a function of the angle of incidence, with either TE or TM polarized radiation. Reflectivity data so recorded have a series of sharp dips corresponding to the excitation of a series of guided modes in the nematic liquid crystal layer (see figure 2(*a*)). (Because of the nature of

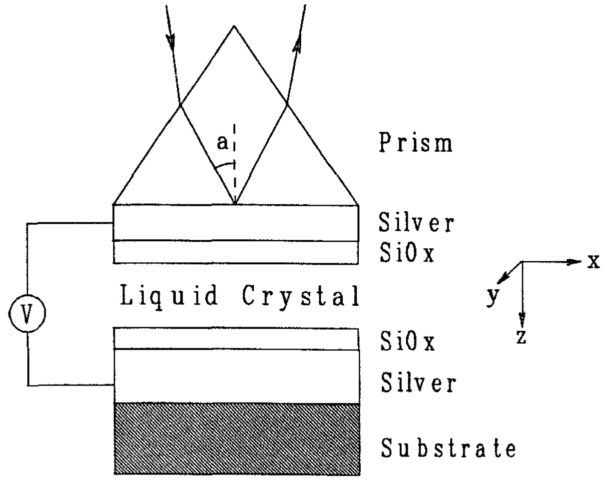


Figure 1. The sample cell construction used in the experiments.

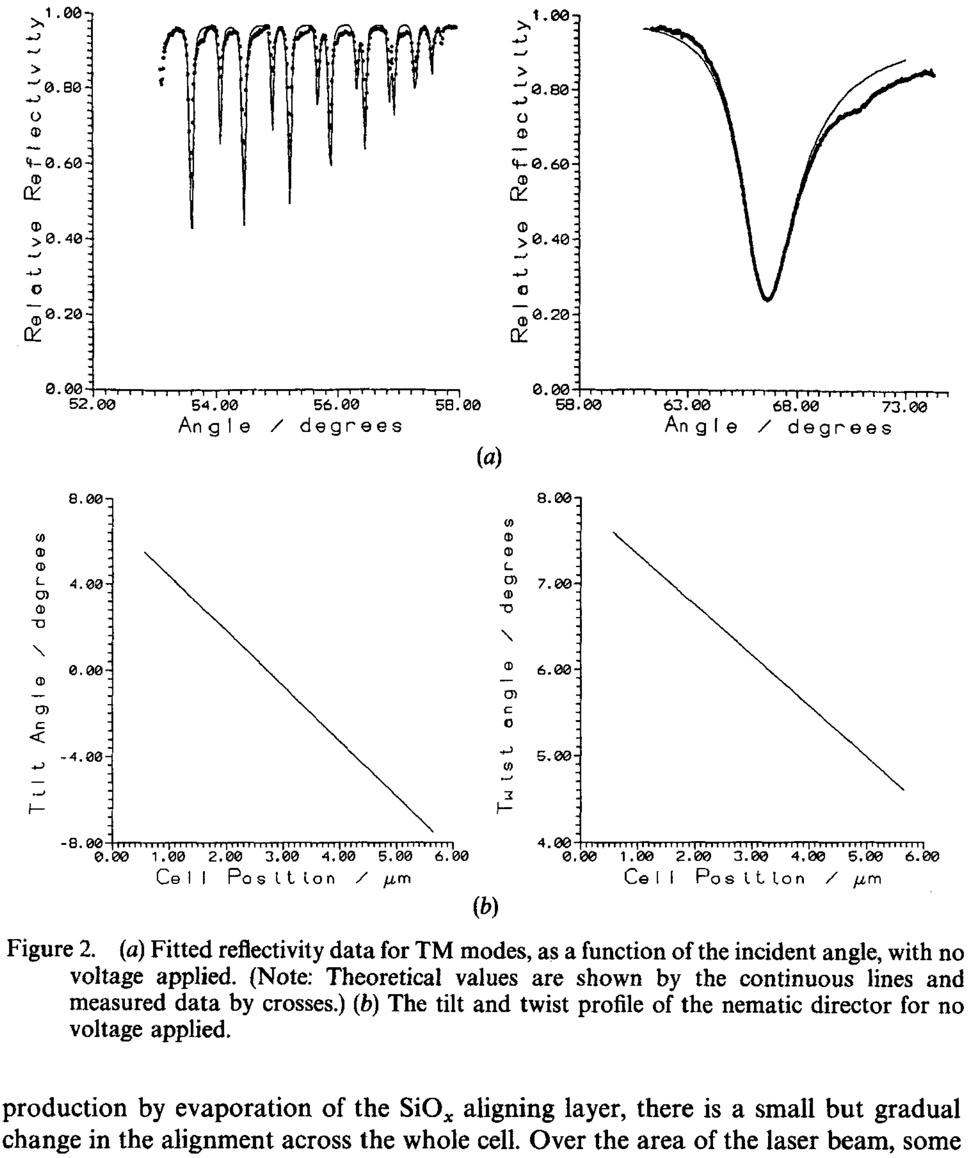


Figure 2. (*a*) Fitted reflectivity data for TM modes, as a function of the incident angle, with no voltage applied. (Note: Theoretical values are shown by the continuous lines and measured data by crosses.) (*b*) The tilt and twist profile of the nematic director for no voltage applied.

production by evaporation of the SiO_x aligning layer, there is a small but gradual change in the alignment across the whole cell. Over the area of the laser beam, some $2\,mm^2$, the spread in alignment was so small as to be negligible.) On applying a low frequency (10kHz) field of up to 10V, the reflectivity data are seen to change substantially (see figure 3(*a*)).

3. Results and discussion

3.1. *Director profile*

It is clearly important to establish the director profile at zero applied field. From the boundary condition produced by the obliquely evaporated SiO_x (for E7 the critical surface tilt angle is ~24° [2]), it is anticipated that the zero voltage state would be either a substantially horizontal (H) state or a twisted vertical (T) state. From the guided mode reflectivity data, it is straightforward to separate these two models. For either TM (optic E field in the incident plane) or TE (optic E field normal to the incident plane)

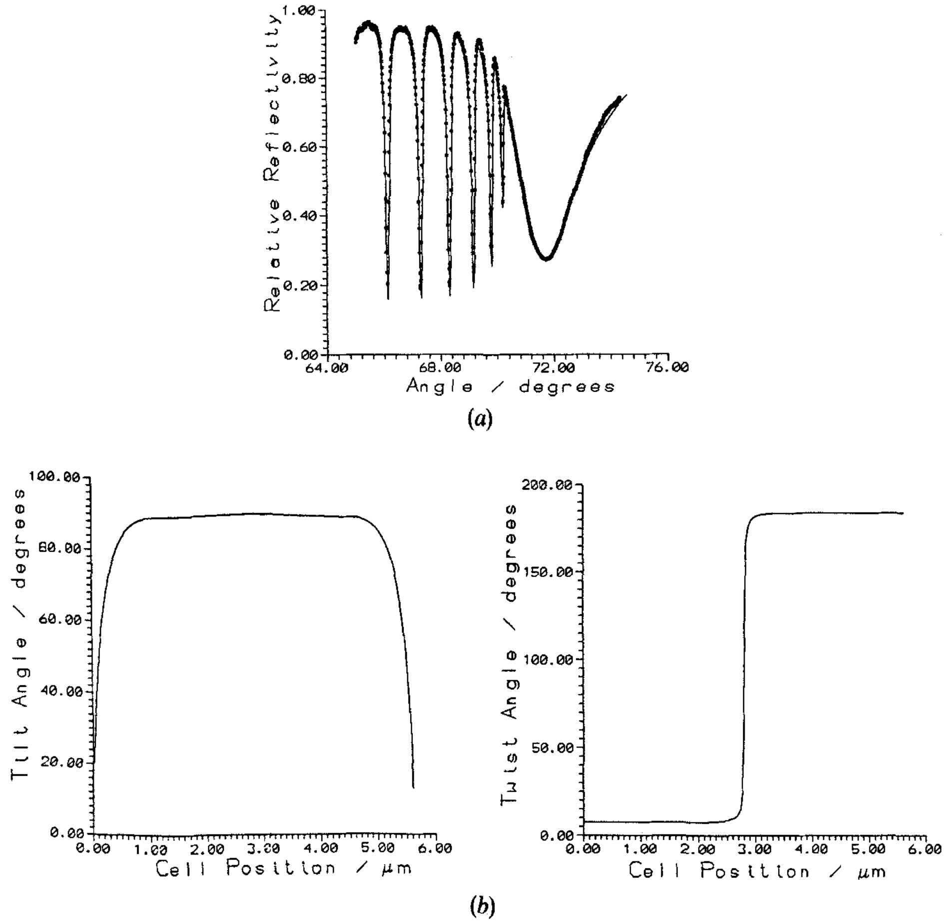


Figure 3. (*a*) Fitted reflectivity data for TM modes, as a function of the incident angle, with an applied field of 9 V across the cell. (Note: Theoretical values are shown by the continuous lines and measured data by crosses.) (*b*) The tilt and twist profile of the nematic director of the twisted vertical state with 9 V applied field.

guided modes, the low order mode positions are determined primarily by the director alignment at the cell centre. This is clearly the case for the lowest order mode where the field maximum is at the cell centre [14]. At zero volts we note that even these lowest order modes, those with greatest momentum in the *xy* plane, all appear at low incidence angles. The lowest order TM guided mode ($\sim 57{\cdot}8°$) is close to the position of the mode found in a previous study [15] using a 90° twisted homogeneous aligned cell of the same material. However by contrast to the 90° twisted cell, there is no evidence for TE-like guided modes present in the surface plasmon-polariton (SPP) dip in the TM reflectivity data. This immediately implies an untwisted state, the horizontal (H) state being the most likely. For this state the TE modes are dictated by $\varepsilon_\perp$ of the liquid crystal at optical wavelengths, that is n_0^2, with the TM modes being also largely dictated by the same $\varepsilon_\perp$. Hence, the lowest order TE modes do not approach in angle the SPP resonance since they are all dictated by $\varepsilon_\perp$ and, according to the metal-clad dielectrical waveguide theory [16], the SPP wavevector is much larger than the cut-off wavevector for either

TE or TM guided modes. Further evidence for the H state of the cell is provided by examining the angular dependent reflectivity for the orthogonal plane of incidence, with the director approximately perpendicular to this plane. Now we expect the TE mode position to be dictated by $\varepsilon_{\parallel}$ ($>\varepsilon_{\perp}$) and hence the cut-off wavevector for the TE modes may be beyond the SPP wavevector. This should give s-like modes in the broad SPP dip—this is observed. Thus the horizontal (H) state of the cell at zero volts is confirmed.

To ascertain the full detail of the director profile at zero volts we need to use Fresnel theory to model the observed angular dependent reflectivity. In order that this may be accomplished, we need first to find values for the two optical permittivities $\varepsilon_{\perp}$ and $\varepsilon_{\parallel}$. This was done by applying a relatively large voltage (~ 10 V) to the cell, forming it into the nearly homeotropically aligned state. By fitting the angular dependent reflectivity for both TE and TM modes at this voltage, we then obtain $\varepsilon_{\perp}$ and $\varepsilon_{\parallel}$ to sufficient precision to allow fitting to the zero voltage data. Continuum theory dictates that, for the small tilt and twist of the cell at zero volts, there will be an almost linear variation of both these variables across the cell. Using Fresnel theory for a multilayer model of the system, the cell thickness, *d*, and surface tilt and twist angle are adjusted until the model fits well with the observed data.

The final fit to the SPP and the guided modes is shown in figure 2(*a*), arising from the tilt and twist profiles of figure 2(*b*) with

$$\varepsilon_{\perp}=2\cdot3160(\pm0\cdot0002)+\mathrm{i}0\cdot0005(\pm0\cdot0002),$$

$$\varepsilon_{\parallel}=2\cdot8740(\pm0\cdot0002)+\mathrm{i}0\cdot0005(\pm0\cdot0002),$$

$$d=5\cdot65(\pm0\cdot01)\,\mu\mathrm{m}.$$

The surface twists, measured with respect to the *x*-axis (see figure 1), are 7·6° and 4·6° on the two surfaces, while the surface tilts, measured with respect to the surface, are 5·5° and $-7\cdot5°$, respectively. This gives a total twist across the cell of $(3\pm0\cdot3)°$ which is due to a combination of misalignment of the two surface directions and small surface director twist at the surfaces associated with the finite surface tilt.

Now consider what happens to this cell under the application of an AC field, remembering that nearly in the centre of the cell, at zero volts, there is a tilt of 0° (horizontal director). From continuum theory there are four possible states [1], two of them are still the plane horizontal states with the position of 0° tilt displaced towards either the upper or lower surface, and the other two are the twisted clockwise or anticlockwise vertical states which may only be switched to through the presence of disclinations, almost inevitably present in these cells.

By examining the angular dependent reflectivity as a function of voltage, we find that, at about 0·6 V, the guided mode structure starts to degrade and on increasing this voltage, the modes gradually broaden and become indistinct. An example of the type of response recorded for these lower voltages is shown for 1 V in figure 4. This is very suggestive of domain formation in the cell and, not surprisingly, these complicated reflectivity data cannot be fitted by any model for one single state of the cell. It implies coexistence of both H and T states or some mixture of them. On further increase in applied field, to about 3 V, the modes sharpen again and, beyond this voltage, a series of very sharp resonances is recorded. This suggests a monodomain and a stable director configuration.

As the voltage is further increased, the TM guided modes move progressively to higher angles and approach the range of the SPP resonance. This is simply because in

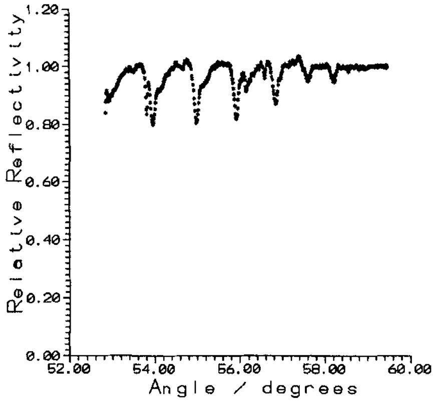


Figure 4. The reflectivity data for TM modes with an applied field of 1 V across the cell.

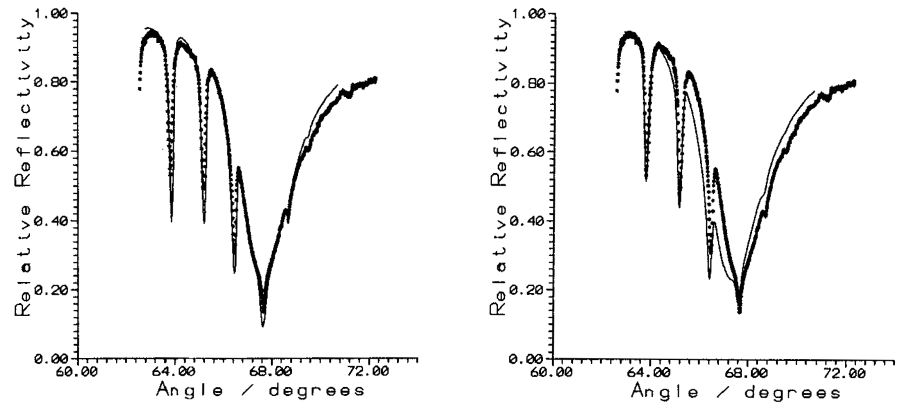
The low frequency (10 kHz) dielectric parameters and elastic contants for E7 at 40°C, used in this calculation.

Parameter	Value
$\varepsilon_{\parallel}$	18·14
$\varepsilon_{\perp}$	5·54
$k_{11}/10^{-12}\,N$	9·0
$k_{22}/10^{-12}\,N$	1·21
$k_{33}/10^{-12}\,N$	1·31

the centre of the cell the director is tending towards homeotropic alignment, which increases the effective ε for the guided modes, towards $\varepsilon_{\parallel}$. Using continuum theory [10], with the values listed in the table of the low frequency (10 kHz) dielectric constants, together with the elastic constants we may model the director profile in the cell. By adjustment of the boundary conditions we may then iteratively converge to the correct director profile which gives, from multilayer optical theory, the angular dependent reflectivities recorded.

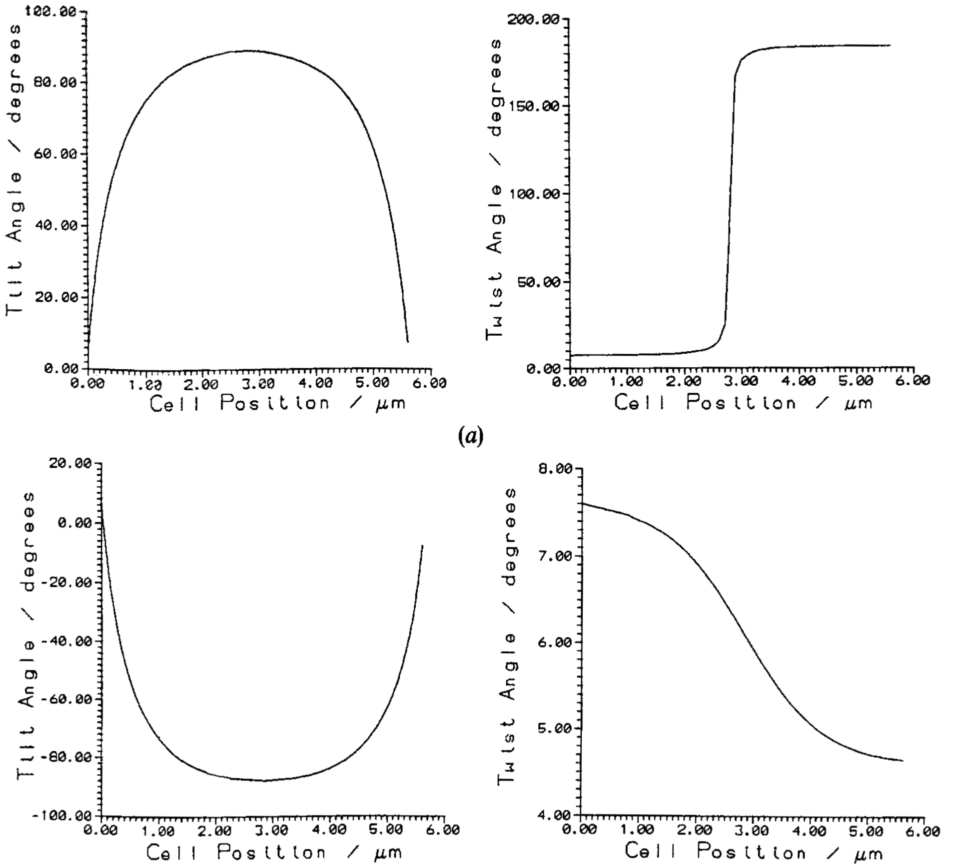
This modelling gives excellent fits to the data (see figure 5 (*a*)) provided we force the continuum theory into the clockwise T state solution. If we insist on an H state with the position of 0° tilt displaced towards the upper surface, then we cannot obtain a good fit (see figure 5 (*b*)) to the data. Likewise the other two possible states do not predict reflectivities in accord with the data. This is not too surprising, since those states have optically distinguished director profiles at the same applied voltage (see figures 6 (*a*) and 6 (*b*)). It is the sensitivity of the optical mode momentum to the details of the director tilt and twist profile that allows the discrimination between these states.

At higher voltages the director profile for the clockwise T state continues to fit the data well (see figure 3 (*a*)). Of course, at these higher voltages, the T state becomes much closer in form to the V state; indeed for a tilt angle maximum greater than about 88°, it is very difficult to differentiate optically between the two models. In contrast, for voltages below 9 V, the clockwise T state gives clearly the correct fit to the data and primarily for this reason we use the same T state description at higher voltages. As is clear for 9 V, the director twist is now almost entirely concentrated in the middle of the cells, while the tilt



(*a*) (*b*)

Figure 5. The reflectivity data for TM modes with an applied field of 3 V, compared with the theoretical curve of (*a*) the clockwise T state, (*b*) the H state with the same tilt and twist on the upper/lower surface as the above T state. (Note: theoretical values are the continuous lines and measured data are the crosses.)



(*b*)

Figure 6. The tilt and twist profile of the director with 3 V applied field for (*a*) the clockwise T state, (*b*) the H state used to generate the theory of figure 5.

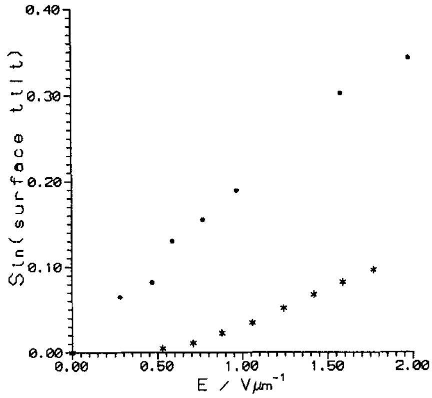


Figure 7. The variation of the surface tilt of the director with applied field, E. The data from this work are shown by the crosses. Data from [17] are shown as the filled circles.

changes are concentrated in thin layers near the surfaces. To obtain the very best fit, it is found necessary to introduce a small voltage dependent surface tilt. This arises because the surface anchoring energy is not infinite [17]. The results for the needed variation of surface tilt with voltage are shown in figure 7. By comparison with the data obtained by Welford and Sambles [17] for an untwisted homogeneously aligned cell, it appears that the surface anchoring energy is higher in the present experiment.

3.2. *Free energy*

It is of interest to establish from a free energy perspective that the T state should be the stable state in an applied field. We use the boundary layer model proposed by Cheng *et al.* [2]. If we have two functions $\phi(z)$, $\theta(z)$ which describe the twist angle ϕ, and tilt angle θ, measured from the xy plane (see figure 1), as functions of the position, z, along the normal to the cell surface, then under an applied field V we have the free energy density

$$F\left(\theta(z),\phi(z),\frac{d\theta}{dz},\frac{d\phi}{dz},z,D\right)=\frac{f(\theta)}{2}\left(\frac{d\theta}{dz}\right)^2+\frac{g(\theta)}{2}\left(\frac{d\phi}{dz}\right)^2+\frac{\mathbf{D}\cdot\mathbf{E}}{8\pi}, \qquad (1)$$

where

$$f(\theta)=k_{11}\cos^2\theta+k_{33}\sin^2\theta$$

$$g(\theta)=(k_{22}\cos^2\theta+k_{33}\sin^2\theta)\cos^2\theta,$$

and D is the electric displacement. Minimization of the integral of F by the standard Euler–Langrange method, followed by integration leads to

$$f(\theta)\left(\frac{d\theta}{dz}\right)^2+g(\theta)\left(\frac{d\phi}{dz}\right)^2-\frac{D^2}{4\pi\varepsilon_\perp(1+\gamma\sin^2\theta)}=A$$

$$g(\theta)\left(\frac{d\phi}{dz}\right)=B, \qquad (2)$$

where A and B are integration constants with

$$\gamma=\frac{\varepsilon_\parallel-\varepsilon_\perp}{\varepsilon_\perp}=\frac{\Delta\varepsilon}{\varepsilon_\perp}.$$

At high fields, the director distortion tends to concentrate in thin layers with most of the cell having the director aligned parallel to the field. Thus we can view the two surfaces as largely decoupled, with each surface layer having in effect a semi-infinite layer adjacent to it [2]. When $\phi=\text{constant}$, the boundary condition $\theta=\pi/2$ and $d\theta/dz=0$ at $z=\infty$ gives in this boundary layer approximation

$$\frac{d\theta}{dz}=\frac{\pm(\varepsilon_{\|}/\varepsilon_{\perp})^{1/2}\cos\theta}{\xi(1+\gamma\sin^2\theta)^{1/2}(1+k\sin^2\theta)^{1/2}},\tag{3}$$

where

$$\xi=\left(\frac{\varepsilon_{\|}}{D}\right)\left(\frac{4\pi k_{11}}{\Delta\varepsilon}\right)^{1/2}$$

and

$$k=(k_{33}-k_{11})/k_{11}.$$

Thus the total energy per unit area, including the elastic energy and the excess of dielectric energy, for each semi-infinite boundary layer is

$$E(\theta_0,\theta_\infty)=\left(\frac{k_{11}}{\xi}\right)q(\theta_0,\theta_\infty),\tag{4}$$

where

$$q(\theta_0,\theta_\infty)=\left(\frac{\varepsilon_{\|}}{\varepsilon_{\perp}}\right)^{1/2}\int_{\theta_0}^{\theta_\infty}\frac{\cos\theta(1+k\sin^2\theta)^{1/2}\,d\theta}{(1+\gamma\sin^2\theta)^{1/2}}\tag{5}$$

and θ_∞ is the value of θ for $z\gg\xi$.

With this simple model, we compare the total energy per unit area of the H state with that of the T state at high voltage. Because, in the T state, the director twist concentrates in the homeotropically aligned cell centre, for which there is no energy contribution, then the boundary layer model may be used with two near-surface regions with no twist. In the horizontal (H) state, which is an asymmetric horizontal state derived from continuum theory, there are three boundary layers (see figure 8 (*a*)), and the total energy per unit area is

$$E_{\mathrm{H}}=E(\theta_{01},\pi/2)+E(0,\pi/2)+E(0,\theta_{02})=2E(0,\pi/2)+E(\theta_{01},\theta_{02})\tag{6}$$

according to equations (4) and (5); here θ_{01} and θ_{02} are the director tilts on the upper and the lower cell surfaces, respectively. By contrast, in the vertical twisted state (T), there are only the two boundary layers (as discussed), see figure 8 (*b*), and the total energy per unit area is then

$$\begin{aligned}E_{\mathrm{T}}&=E(\theta_{01},\theta_{\mathrm{m}})+E(\theta_{02},\theta_{\mathrm{m}})+E_{\mathrm{TWIST}}\\&=2E(0,\theta_{\mathrm{m}})+E(\theta_{01},\theta_{02})-2E(0,\theta_{02})+E_{\mathrm{TWIST}},\end{aligned}\tag{7}$$

where E_{TWIST} is the twist energy, θ_{m} is the largest tilt angle. In the high voltage limit $\theta_{\mathrm{m}}\to\pi/2$ and as this occurs $E_{\mathrm{TWIST}}\to 0$ [18].

Hence, comparing with equation (6), we see that $E_{\mathrm{H}}>E_{\mathrm{T}}$ on the basis of this simple model. Of course this agreement is only valid above a few volts, where experimentally we have shown exactly this behaviour, i.e. that the T state is stable relative to the H state.

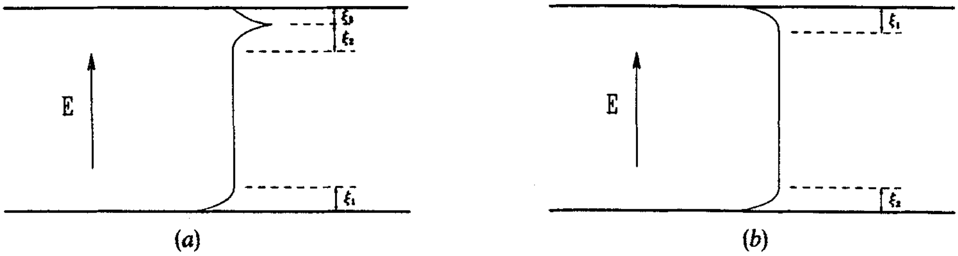


Figure 8. The nematic configuration of the director tilt in (*a*) the H state, and (*b*) the clockwise T state under an applied field.

4. Conclusions

Using the optical excitation of guided modes in a nematic cell with oppositely directed surface tilts we have studied the equilibrium state as a function of applied field. At zero volts we find that the stable configuration is the horizontal state with surface tilt angles of 5·5° and −7·5°, measured from the cell surfaces, respectively, for 74° (with respect to the surface normal) evaporated SiO_x aligning layers.

Under application of a field, the horizontal state, in the presence of defects, renucleates into a twisted vertical state, as evidenced by a confused form to the optical mode excitation monitored by the angle dependent reflectivity. Above about 3 V, the twisted vertical state is fully stable and all data at this and higher voltages fit the predictions of a model based on this state. To model the data fully, it is necessary to introduce a surface tilt which changes with voltage, in this case of the order 0·67° V^{-1}, indicating stronger anchoring than that for 60° evaporated SiO_x. The preference of the finite surface tilt cell for the T state is also confirmed by a simple free energy calculation based on the boundary layer model.

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References

[1] Raynes, E. P., 1975, *Rev. Phys. Appl.*, **10**, 117.
[2] Cheng, J., Thurston, R. N., and Bereman, D. W., 1981, *J. appl. Phys.*, **52**, 2756.
[3] Bos, P. J., and Beran, K. R., 1984, *Molec. Crystals liq. Crystals*, **113**, 329.
[4] Cheng, J., and Thurston, R. N., 1981, *J. appl. Phys.*, **52**, 2766.
[5] Thurston, R. N., 1981, *J. appl. Phys.*, **52**, 3040.
[6] Welford, K. R., and Sambles, J. R., 1989, *Liq. Crystals*, **2**, 91.
[7] Elston, S. J., and Sambles, J. R., 1989, *Appl. Phys. Lett.*, **55**, 1621.
[8] Elston, S. J., and Sambles, J. R., 1991, *Molec. Crystals liq. Crystals*, **200**, 167.
[9] Elston, S. J., 1991, *Liq. Crystals*, **9**, 769.
[10] Deuling, H. J., 1974, *Molec. Crystals liq. Crystals*, **27**, 81.
[11] Ko, D. Y. K., and Sambles, J. R., 1988, *J. opt. Soc. Am.* A, **5**, 1863.
[12] Kretschmann, E., 1971, *Z. Phys.*, **241**, 313.
[13] Cognard, J., 1981, *Molec. Crystals liq. Crystals*, **78**, 1.
[14] Tien, P. K., 1971, *Appl. Opt.*, **10**, 2395.
[15] Ruan, L. Z., Elston, S. J., and Sambles, J. R., 1991, *Liq. Crystals*, **10**, 369.
[16] Kaminow, I. P., Mammel, W. L., and Weber, H. P., 1974, *Appl. Optics*, **13**, 396.
[17] Welford, K. R., and Sambles, J. R., 1987, *Appl. Phys. Lett.*, **50**, 871.
[18] Preist, T. W., Welford, K. R., and Sambles, J. R., 1989, *Liq. Crystals*, **4**, 103.